#### **TECHNICAL ARTICLE**



# A New Risk Assessment Model for Underground Mine Water Inrush Based on AHP and D–S Evidence Theory

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#### Abstract

Effective risk assessment of underground water inrush is the prerequisite and basis for mine water hazard control and safe mining. An inrush risk assessment system was set up, based on a comprehensive analysis of the factors of mine water inrush risk and improved analytic hierarchy process (AHP). A new judgment matrix was constructed based on a scale of 1–9. The Dempster-Shafer (D–S) synthetic rule was improved on the basis of improved AHP; the frame of discernment proposed includes water-inrush, critical condition, and no water-inrush. A water-inrush integration decision-making model was thus established. Finally, using a typical underground mine in China, the new model was verified using this method. The probability of water inrush was 0.822, which is broadly in line with the actual situation, indicating that the model is feasible and applicable.

**Keywords** Water-inrush integration decision-making model  $\cdot$  Improved AHP  $\cdot$  Improved D–S evidence theory  $\cdot$  Frame of discernment  $\cdot$  Manmade factors

#### Introduction

In recent years, the underground mine water inrushes accidents occurred frequently. There were 146 water inrush accidents from 2007 to 2016 in China, which resulted in 786 deaths (Table 1). Thus, scientific and effective assessment of underground mine water inrush risks is important.

The analytic hierarchy process (AHP), combined with other methods, has been widely used to assess the risk of underground mine water inrush. A multivariate fuzzy assessment model was used to evaluate the risk of floor water invasion in coal mines based on a fuzzy comprehensive

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assessment and AHP (Wang et al. 2012). In addition, an attribute synthetic assessment system was combined with attribute mathematical theory and AHP to assess the risk of floor water inrush in coal mines (Li et al. 2014). An assessment index system and assessment standard were established for coal floor water inrush based on conditions of the aquifer and aquiclude, geologic structure, and mining disturbance (Li and Chen 2016), who used Grey relational analysis (GRA) and AHP to establish an assessment model that effectively overcomes the uncertainty between the indices of water inrush effects and really reflects the degree of importance for each of those indices. In addition, fuzzy Delphi analytic hierarchy process (FDAHP) was combined with GRA to predict and prevent water inrush (Qiu et al. 2017). AHP was coupled with GIS to evaluate the water abundance of the roof aquifer to predict and prevent water inrushes (Wu et al. 2013; Zeng et al. 2017).

MATLAB, a mathematics software package with the high performance of a numerical algorithm, is commonly used to calculate the matrix algebra in AHP. Based on MATLAB, a back-propagation (BP) neural network was used to determine mine water inrush sources, with improved assessment accuracy in practical projects (Li and Zhang 2011). The risk of water inrush from a coal floor occurred was predicted accurately based on a small sample using MATLAB (Yang



**Table 1** Underground mine water accidents from 2007 to 2016 in China

Year	Accidents times	Death toll		
2007	27	142		
2008	20	105		
2009	22	109		
2010	17	104		
2011	16	68		
2012	10	72		
2013	11	61		
2014	9	41		
2015	8	55		
2016	6	29		

et al. 2012). A risk assessment system, which is reliable and feasible, was also established for coal seam floor water inrush based on Visual Basic (VB) and MATLAB (Li 2015; Liu et al. 2015).

Dempster-Shafer (D–S) evidence theory was also applied to identify the sources of underground mine water inrush. An evaluation model for mine water inrush prediction was proposed base on D–S evidence theory and data collected by multi-sensor (Zhang and Cheng 2008). Coupled with neural networks, D–S evidence theory was used to build a forecast model of water-inrush from the mine floor (Ma and Zhang 2012).

The natural geological factors influencing underground mine water are various (Chai and Yao 2014; Jin and Wang 1988; Min 2014); nevertheless, some factors, especially manmade factors (i.e. preventive works and safety management), are not considered in existing methods. At the same time, the shortcomings of AHP and D–S are rarely

considered. The original AHP judgment matrix is constructed by individual decision-makers, resulting in great variations, or even error. Also, the D–S synthetic rule is meaningless in some special conditions (for example, when the two mass sets completely conflict, as detailed later). What's more, AHP is rarely combined with D–S for water inrush risk assessment, even though they were used to handle dependence among human error in human reliability analysis (Su et al. 2015). Therefore, mine water inrush risk factors (including manmade factors) were analyzed comprehensively in this paper, and AHP and D–S were improved to evaluate the risk of mine water inrush more accurately and effectively.

### **Hydrogeological and Geological Conditions**

In this study, the 20,101 ventilation drift in the Wangjialing no. 2 coal mine, located in Shanxi Province in China, was used as a research site (Fig. 1). The whole region is stable and recoverable, the coal seam is 2.98 to 8.50 m thick, with an average thickness of 6.17 m. The mine generally contains 1 to 2 layers of sandstone, with up to 5 layers in some locations. The thickness of the sandstone layers is typically about 0.20 m, with a few reaching 1.00 m (no. 18 hole). The roof of the recoverable coal layer is mostly mudstone and siltstone. Most of the bottom plate is siltstone and mudstone, while some parts are fine sandstone and quartz sandstone. The water inflow is 0.066-0.91 L/(s·m) of drift (the water pressure is 0-2.02 MPa, the thickness of the aguitard is 13.04–59.93 m, and the water inrush coefficient is 0.004–0.06 MPa/m). It should be noted that the water inrush coefficient is the ratio of the hydrostatic pressure in

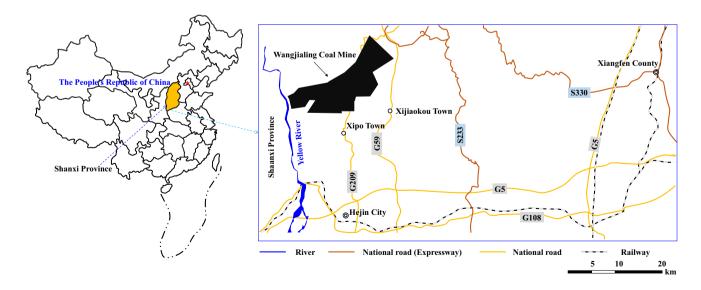


Fig. 1 Location of the Wangjialing coal mine

the aquifer to the thickness of the aquifuge, which is used to characterize the danger of a mine water disaster (Liu 2009). The thickness of the overlying layer is more than 180 m and the height of the water-conducting fracture zone is about 54 m. Apart from the surface water in the eastern area of the mine that may have influenced exploitation, atmospheric precipitation, and surface water will generally not affect mining.

The inrush potential for the no. 2 coal seam is mainly affected by fissure water in the sandstone roof, Taiyuan limestone aquifer, and small abandoned underground workings. After taking certain exploration and drainage technical measures (including detailed exploration before mining, cover grouting, borehole drains, and roof drainage), the management of the Wangjialing coal mine thought that the aquifer should not pose a threat to mining under normal circumstances, and that prevention and treatment of the underground mine water should focus on nearby small abandoned underground workings. Although a large amount of investigation and exploration has been carried out recently, the need still exists to determine the potential threat due to exploitation in their vicinity.

#### **Methods**

### The AHP Method and its Improvement

#### The AHP Method

The AHP method, which combines qualitative analysis with quantitative analysis, was developed by Thomas L. Saaty (1980). It has been widely applied in the mining field. The specific AHP processes are: (1) determine the hierarchy between the indices and construct the AHP structure.

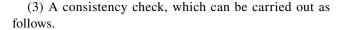
(2) Construct the following judgment matrix *A* using the 1–9 scales (Saaty 1990).

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$
 (1)

where  $a_{mn}$  is the result of the comparison of the importance of mth factor and the nth factor,  $a_{mn} > 0$ ,  $a_{mn} = 1/a_{nm}$ ,  $a_{mn} = 1$  (when m = n).

**Table 2** The random consistency index for  $1 \sim 15$  assessment





$$C.R = \frac{C.I}{R.I} \tag{2}$$

where *C.R* is the random consistency ratio, *C.I* is the consistency index of the judgment matrix, and *R.I* is the random consistency index. The consistency index of the judgment matrix can be further specified as,

$$C.I = \frac{\lambda_{\text{max}} - n}{n - 1} \tag{3}$$

where  $\lambda_{\rm max}$  is the maximum eigenvalue of A; n is the quantity of the assessment indices. Repeat calculating the random judgment matrix eigenvalues for 500 times or more and then calculate the arithmetical average of the eigenvalues, which is the random consistency index. The random consistency index for 1–15 assessment indices after 1000 iterations have been calculated by Xu (1988) (Table 2).

(4) Using the hierarchical ranking, determine the factors ranking.

# The Shortcomings in the Construction of the Judgment Matrix and its Improvement

The original judgment matrix is constructed by individual decision-makers, and the accuracy of the decision-making is not always sufficient for practical purposes due to the lack of individual subjective knowledge and dependence of empiricism. To improve the construction of the judgment matrix, the Delphi method was used, which takes full advantage of each expert's opinion. At the same time, the average judgment matrix method was used to obtain the unique judgment matrix, as the judgment matrices constructed by each expert are different. In this method, the judgment matrix is constructed by assessment experts based on the Delphi method and 1–9 scales method, then the average of the judgment matrices is calculated, and finally the unique judgment matrix is obtained (Li and Zhang 2004). The specific procedure of the new method is as follows:

Prepare assessment information for experts. (2) Contact assessment experts and form expert group. (3) Assemble supplemental assessment information. (4) Each expert determines the initial judgment matrix based on Delphi method and 1–9 scales method. (5) Summarize and analyze the initial judgment matrices, and then give feedback to each



expert. (6) Each expert modifies the judgment matrix based on the feedback information. (7) Summarize and analyze the initial judgment matrices again. Repeat the above procedures until each expert does not modify his (her) own judgment matrix. Finally, the unique judgment matrix will be obtained with the average judgment matrix method, which is the judgment matrix in improved AHP.

# The Shortcomings of the Consistency Index and its Improvement

As each expert has a different understanding of assessment information, the subjective judgment is sometimes different from objective reality. The consistency indices of AHP and fuzzy AHP are as follows respectively (Saaty 1980; Chen and Zhao 2004).

$$C.I = \frac{\lambda_{\max} - n}{n - 1}, \ I(R, W) = \frac{FC(R, W)}{n(n - 1)}$$
 (4)

where FC(R, W) is the degree of compatibility between the fuzzy complementary matrix R and its characteristic matrix W,  $FC(R, W) = \sum_{i=1}^{n} \sum_{j=1}^{n} \left| r_{ij} - w_{ij} \right|$  (Yang and Qiu 2010).

Through comparison and analysis, it is clear that the factors affecting the consistency of the judgment matrix are not exactly the same. Therefore, this paper presents an improved consistency index based on the two above indices, as follows:

$$C.I = \frac{2(\lambda_{\text{max}} - n)}{n(n-1)} \tag{5}$$

Systematic errors can be avoided as much as possible with a comparison between every two indices by n(n-1)/2 times, and the negative influence of individual judgment will be reduced at the same time. Although the improved consistency index increases the work, it can provide more information for decision-makers and the probability of error will be reduced through repeated comparisons from different contributors. The improved method does not require the judgment matrix to be completely consistent, but rather only requires that each comparison be relatively independent, so that the accuracy of the final decision is improved as much as possible.

#### The D-S Evidence Theory and its Improvement

#### The D-S Evidence Theory

The Dempster–Shafer (D–S) evidence theory deals with the problem of uncertainty (Dempster 1967; Shafer 1976). The D–S evidence theory combines the advantages of conventional decision analysis theory and Bayesian probability theory, in that it not only emphasizes the objectivity of evidence

but also attaches importance to the subjectivity of evidence estimation. Since the necessary prior data can be obtained easily and the data or knowledge of different experts or data sources can be synthesized by the D–S synthetic rule, D–S evidence theory is widely used in the fields of multi-source information fusion, expert systems, and so on.

**Definition 1** Recognition Framework. Assuming we need to judge the problem A,  $\Omega$  is used to describe all the answers to problem A and all the elements in  $\Omega$  are mutually exclusive. We call the  $\Omega$  is the recognition framework.  $\Omega = \{A_1, A_2, ..., A_k\}$ .

**Definition 2** Basic Probability Assignment (BPA). In the recognition framework, the basic probability assignment is a so-called mass function that meets the rule, as in (4) and (5):

$$m(\phi) \neq 0 \tag{6}$$

$$\sum_{A_1 \dots A_n \subset \Omega} m(A) = 1 \tag{7}$$

**Definition 3** The D-S Synthetic Rule. The synthetic rule of a limited number of mass functions  $m_1$ ,  $m_2...m_n$ , as in formula (8) and (9).

$$m(A) = \begin{cases} \frac{\sum_{A_1 \dots A_n \subset \Omega} m_1(A_1) \dots m_n(A_n)}{A_1 \cap \dots \cap A_n = A}, \forall A \subset \Omega, A \neq \Phi \\ 0, A = \Phi \end{cases}$$
(8)

$$c = \sum_{\substack{A_1 \dots A_n \subset \Omega \\ A_1 \cap \dots \cap A_n \neq \Phi}} m_1(A_1) \dots m_n(A_n)$$
(9)

where c is defined as the conflict factor which is a measure of the amount of conflict between the two mass sets.

# The Shortcomings of the D–S Synthetic Rule and its Improvement

When c=1, that is to say, the two mass sets are complete conflict, the D-S synthetic rule (8) is meaningless. When  $c\rightarrow 1$ , that is to say, the two mass sets are high conflict, the D-S synthetic result disagrees with the reality. Many research studies have been conducted to improve the effectiveness of multi-source information fusion (Yager 1987; Murphy 2000; Haenni 2002). Based on the work of Haenni (2002), the source of the evidence was improved in this study. That is because the weight of evidence provided by different sources of evidence is different from that provided by the synthesis rule, so the weight



coefficient  $w_i$  determined by improved AHP is introduced to describe its importance. The greater the value of  $w_i$ , the more important of evidence and vice versa.

Based on the D–S synthetic rule (8) and the weight coefficient provided by improved AHP, we set  $W = (w_1, w_2, ..., w_n)$ , in which  $w_i \in (0,1)$  and  $\sum_{1}^{n} w_i = 1$ . And define  $\partial_i$  as the discount factor of  $w_i$ ,  $1 - \partial_i = w_i / w_{\text{max}}$ , then we can obtain a new weight vector,  $W = (w_1, w_2, ..., w_n) / w_{\text{max}}$ . The belief functions of the D–S evidence theory were improved as follows.

$$m'(A) = (1 - \partial_i)m(A)$$
  

$$m'(\Theta) = (1 - \partial_i)m(\Theta) + \partial_i$$
(10)

By substituting the belief functions (10) into (8), we will get a new synthesis rule:

$$m(A) = \begin{cases} \sum_{\substack{A_1 \dots A_n \subset \Omega \\ A_1 \cap \dots \cap A_n = A}} (1 - \partial_i)^n m_1(A_1) \dots m_n(A_n) \\ \hline 1 - c \ \forall A \subset \Omega, A \neq \Phi \\ 0 \qquad A = \Phi \end{cases}$$
(11)

where 
$$c = \sum_{\substack{A_1 \ ... \ A_n \subset \Omega \\ A_1 \ ... \ \cap A_n \neq \Phi}} (1 - \partial_i)^n m_1(A_1)...m_n(A_n).$$

on the principles of the risk assessment index system (Chen et al. 2012), a risk assessment system of underground mine water inrush was established (Fig. 2). The target layer in the index system is a risk of mine water inrush, including seven first-grade indices and each of them has corresponding second-grade indices. Let each first-grade index denoted by  $C_{ij}$ , and each second-grade index denoted by  $B_i$ .

# **Application and Results**

The new model was applied in the 20,101 ventilation drift of the Wangjialing no. 2 coal mine.

#### Weight Calculation by the Improved AHP

Based on the improved AHP, the weight of each index can be calculated. First, 20 assessment experts were asked to construct a judgment matrix  $B_{Ik}$  (k means No. k expert, k=1,2,...,k,..., 20) based on the 20,101 ventilation drive in Wangjialing coal mine using the Delphi method. Then, we calculated the unique judgment matrix B with the average judgment matrix method. B is the judgment matrix in improved AHP.

$$B = \begin{pmatrix} 1a_{12} \cdots a_{1n} \\ a_{21} 1 \cdots a_{2n} \\ \vdots \vdots \vdots \\ a_{n1} a_{n2} \cdots 1 \end{pmatrix} = \begin{pmatrix} 12 & 40/_{11} & 30/_{11} & 8/_3 & 40/_{11} & 20/_{11} \\ 1/_{21} & 20/_{11} & 20/_{11} & 3/_2 & 30/_{11} & 5/_{11} \\ 11/_{40} & 11/_{20} 1 & 20/_{11} & 10/_{33} & 20/_{11} & 5/_{22} \\ 11/_{30} & 11/_{20} & 11/_{20} 1 & 20/_{11} & 20/_{11} & 10/_{11} \\ 3/_{8} & 2/_{3} & 33/_{10} & 11/_{20} 1 & 5/_{21} & 22/_{83} \\ 11/_{40} & 11/_{30} & 11/_{20} & 11/_{20} & 21/_{51} & 9/_{8} \\ 11/_{20} & 11/_{5} & 22/_{5} & 11/_{10} & 83/_{22} & 8/_{9}1 \end{pmatrix}$$

$$(12)$$

## **Risk Assessment of Underground Mine Water Inrush**

Mine water-inrush disasters in China are influenced by various factors, and their formative conditions are also different. After decades of water control projects and researchers' continuous exploration, a good understanding of the water-inrush-related factors and their impact has been obtained. In summary, the conditions leading to mine water inrush include congenital, natural geological and manmade conditions.

The natural geological factors influencing underground mine water can be summarized as five aspects: (1) water inrush sources, (2) water inrush coefficient, (3) water discharge capacity, (4) underground watercourse, and (5) the condition of the roadway. The manmade factors influencing underground water can be summarized as: (1) preventive works and (2) safety management. Based

As there are many judgment matrices in this paper and the dimension of each matrix is large, mathematical software MATLAB was used for the calculations. The matrix  $\boldsymbol{B}$  was normalized as  $\overline{\boldsymbol{B}}$  by columns, and then the greatest characteristic root  $\lambda_{\max}$  and characteristic vector  $\boldsymbol{W}$  was calculated.

$$\lambda_{\text{max}} = 8.1447 \tag{13}$$

 $W = (0.2659, 0.1426, 0.0876, 0.1075, 0.0870, 0.1144, 0.1950)^{T}$ (14)

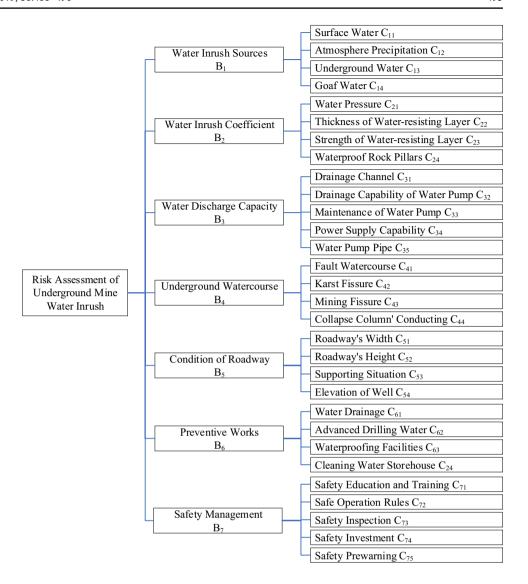
The value of each item in W is the weight each first-grade index to the target layer. Then a consistency check can be carried out.

$$C.I = \frac{2(\lambda_{\text{max}} - n)}{n(n-1)} = \frac{2(8.1447 - 7)}{7 \times 6} = 0.0545$$
 (15)

$$C.R = C.I/R.I = 0.0545/1.36 = 0.0401 < 0.10$$
 (16)



Fig. 2 Risk assessment system for an underground mine water inrush



When C.R < 0.1, the consistency of the judgment matrix is considered as satisfactory, which means the weight distribution is reasonable. Otherwise, the matrix must be adjusted until consistency is satisfied. Then it is easy to find that  $\boldsymbol{B}$  is of good consistency.

In summary, the weight of each first-grade to the target layer is:  $B_1 = 0.2659$ ,  $B_2 = 0.1426$ ,  $B_3 = 0.0876$ ,  $B_4 = 0.1075$ ,  $B_5 = 0.0870$ ,  $B_6 = 0.1144$ ,  $B_7 = 0.1950$ . They thus rank as:  $B_1$  (water inrush sources) >  $B_7$  (safety management) >  $B_2$  (water inrush coefficient) >  $B_6$  (preventive works) >  $B_4$  (water discharge capacity) >  $B_5$  (condition of the roadway) >  $B_3$  (underground watercourse).

The weight of each second-grade index can be calculated with the same process, and then the weight of each second-grade index to the target layer can be calculated with formula (17) (Table 3).

$$w_{2-0} = w_{2-1} \times w_{1-0} \tag{17}$$

where  $w_{2-0}$  is the weight of each second-grade index to the target layer,  $w_{2-1}$  is the weight of each second-grade index to the first-grade, and  $w_{1-0}$  is the weight of each first-grade to the target layer.

From Table 3, it can be seen that the five most weighted second-grade indices are  $C_{14}$  (goaf water),  $C_{24}$  (waterproof rock pillars),  $C_{75}$  (safety pre-warning),  $C_{13}$  (underground water),  $C_{44}$  (collapse column conducting). Although  $w_{2-1}$  ( $C_{24}$ ),  $w_{2-1}$  ( $C_{44}$ ),  $w_{2-1}$  ( $C_{62}$ ) are greater than  $w_{2-1}$  ( $C_{14}$ ),  $w_{2-0}$  ( $C_{14}$ ) is the greatest among  $w_{2-0}$  ( $C_{ij}$ ). This is because  $w_{1-0}$  ( $B_1$ ) is much greater than other  $w_{1-0}$  ( $B_i$ ), indicating that the  $B_1$  (water inrush sources) is more weighted than other first-grade indices.

#### **BPA and D-S Synthetic**

Based on the analysis of water-inrush factors, the frame of discernment  $\Omega$  was proposed as  $\Omega = \{A_1, A_2, A_3\} = \{\text{water-inrush}, \text{critical condition, no water-inrush}\} = \{a, b, c\}$ . Based



**Table 3** The weights and BPA of each index

First-grade index	<i>w</i> <sub>1-0</sub>	Second-grade index	$w_{2-1}$	$w_{2-0}$	Improved BPA			
					m (a)	m (b)	m (c)	m (θ)
$B_1$	0.2659	C <sub>11</sub>	0.0336	0.1264	0.032	0.084	0.042	0.842
		$C_{12}$	0.0178	0.0668	0.022	0.033	0.045	0.900
		C <sub>13</sub>	0.0712	0.2677	0.067	0.112	0.223	0.598
		$C_{14}$	0.1433	0.5391	0.540	0.225	0.090	0.145
$\mathbf{B}_2$	0.1426	$C_{21}$	0.0301	0.2113	0.063	0.110	0.126	0.701
		C <sub>22</sub>	0.0186	0.1307	0.068	0.058	0.058	0.815
		C <sub>23</sub>	0.0076	0.0533	0.032	0.016	0.032	0.921
	C <sub>24</sub>	0.0862	0.6047	0.540	0.270	0.090	0.100	
$B_3$	0.0876	C <sub>31</sub>	0.0147	0.1682	0.073	0.128	0.146	0.654
		C <sub>32</sub>	0.0076	0.0866	0.056	0.056	0.056	0.831
		C <sub>33</sub>	0.0041	0.0470	0.041	0.010	0.020	0.929
		C <sub>34</sub>	0.0248	0.2829	0.245	0.061	0.245	0.448
	C <sub>35</sub>	0.0364	0.4154	0.630	0.090	0.090	0.190	
$B_4$	0.1075	$C_{41}$	0.0296	0.2751	0.043	0.151	0.043	0.763
		$C_{42}$	0.0107	0.0999	0.063	0.047	0.031	0.859
		$C_{43}$	0.0055	0.0513	0.036	0.024	0.016	0.924
		C <sub>44</sub>	0.0617	0.5736	0.720	0.045	0.090	0.145
$B_5$	0.0870	C <sub>51</sub>	0.0451	0.5185	0.630	0.045	0.090	0.235
		$C_{52}$	0.0085	0.0980	0.060	0.051	0.043	0.847
		C <sub>53</sub>	0.0053	0.0607	0.042	0.011	0.042	0.905
		C <sub>54</sub>	0.0281	0.3229	0.336	0.168	0.056	0.440
$B_6$	0.1144	C <sub>61</sub>	0.0324	0.2830	0.181	0.158	0.045	0.616
		C <sub>62</sub>	0.0645	0.5640	0.360	0.135	0.045	0.460
		C <sub>63</sub>	0.0058	0.0510	0.033	0.008	0.033	0.927
		C <sub>64</sub>	0.0137	0.1200	0.077	0.019	0.077	0.828
$\mathbf{B}_7$	0.1950	$C_{71}$	0.0147	0.0756	0.033	0.058	0.066	0.008
		C <sub>72</sub>	0.0421	0.2160	0.190	0.142	0.119	0.024
		C <sub>73</sub>	0.0403	0.2069	0.227	0.045	0.091	0.091
		C <sub>74</sub>	0.0178	0.0914	0.080	0.020	0.080	0.020
		C <sub>75</sub>	0.0800	0.4101	0.630	0.180	0.045	0.145

Bold values are the five most weighted second-grade indices

on the D–S evidence theory, set  $\beta$  as the degree of support that  $C_{ij}$  belongs to  $A_k$ , which is generally 0.9.

For the most-weighted basic probability function

$$m_i(A_i) = \beta \times C_{ij} \tag{18}$$

For the least-weighted basic probability function

$$m_i(A_i) = \frac{w_j}{w_i} \beta \times C_{ij} \tag{19}$$

**Table 4** The BPA of each second-grade index

First-grade index	Weight of first- grade index	Original BPA				Improved BPA			
		m (a)	m (b)	m (c)	m (θ)	m (a)	m (b)	m (c)	m (θ)
$\overline{B_1}$	0.2659	0.380	0.200	0.116	0.305	0.342	0.180	0.104	0.374
$\mathrm{B}_2$	0.1426	0.385	0.228	0.099	0.288	0.186	0.110	0.048	0.656
$B_3$	0.0876	0.602	0.074	0.112	0.213	0.422	0.051	0.078	0.449
$\mathrm{B}_4$	0.1075	0.558	0.067	0.072	0.303	0.203	0.024	0.026	0.746
$B_5$	0.0870	0.596	0.118	0.058	0.229	0.175	0.035	0.017	0.773
$B_6$	0.1144	0.492	0.055	0.065	0.389	0.190	0.021	0.025	0.763
<b>B</b> <sub>7</sub>	0.1950	0.598	0.100	0.036	0.265	0.395	0.066	0.024	0.515



**Table 5** The synthesis results

Synthesis results	BPA						
	m (a)	m (b)	m (c)	m (θ)			
Probability	0.815	0.037	0.014	0.134			

First, the risk assessment of the index layer should be carried out. Experts were invited to ascertain the preliminary BPA, which were improved using formulas (18) and (19). The improved BPA of  $C_{ij}$  (Table 3) was synthesized by the D–S synthetic rule. The results of the risk assessment of the guideline layer are shown as the original BPA in Table 4. The original BPA were improved using formulas (18) and (19) and the improved BPA of  $B_i$  are also shown in Table 4.

The improved BPA in Table 4 was synthesized using the D–S synthetic rule. The results of the risk assessment of the target layer are shown as the BPA in Table 5, which indicates that the probability of water inrush (improved) in the 20,101 ventilation drive in Wangjialing coal mine reaches a peak of 0.822, which is much higher than the probability obtained by the original method (0.645). In fact, the probability of no water inrush [m(c)] fell to 0.016. This result is in accordance with reality (an inrush accident had happened), which demonstrates that the improved approach for risk assessment of underground mine water inrush using AHP and D–S evidence theory was effective and reliable in this case.

#### **Conclusions**

The factors that cause mine water inrush factors are complicated, their dimensions are different, and some of them are qualitative while others are quantitative. To accommodate all of the factors, the D–S evidence theory was augmented with improved AHP and applied to the identification and risk assessment of underground mine water inrush. This provides technical and theoretical support for technical staff to make a decision on whether water inrush in the working floor of mine openings will occur.

- (1) More attention was paid to manmade factors, with natural geological factors also considered. Using this approach, a risk assessment system of underground mine water inrush was established with AHP, and the weight of each index obtained with improved AHP, which laid the foundation for follow-up D-S evidence theory analysis.
- (2) Based on the D-S synthetic rule and the weight coefficient provided by AHP, the D-S synthetic rule was improved.

(3) A risk assessment of water inrush in the 20,101 ventilation drift in the Wangjialing coal mine was carried out and the probability of water inrush is 0.822, which is in accordance with reality (an inrush accident had happened). The credibility is much higher than with the previous methods of fuzzy assessment. It indicates that the improved approach for risk assessment of underground mine water inrush in the 20,101 ventilation drift is effective and can be used for the prevention and treatment of underground mine water. Although the new model was applied and validated in this case, there is still some work to be done to improve the method. Future work should focus on the rapid construction of the judgment matrix as well as application and extension of the method.

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